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Artificial control of the laminar–turbulent transition of a two-dimensional wake by external sound

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Artificial acceleration and deceleration of the transition process in a two-dimensional wake were attempted. The wake was produced behind a thin aerofoil placed parallel to uniform flow. The sound from a loudspeaker introduced into the wake acted as an artificial disturbance. Various kinds of sound were tested and the effect on the transition was judged by the energy spectrum. Sinusoidal sound of the frequency of the maximum growth rate in the linear region decelerates the transition, whereas sound of a different frequency accelerates it. Sound of two or four frequencies is more effective in accelerating the transition when the frequencies are properly chosen. White noise from the loudspeaker is not effective, but a two-peak noise specially designed for strong nonlinear interaction is the most effective in accelerating the transition process. These results can be explained by two empirical properties of the nonlinear interaction: the growth suppression induced by a large amplitude fluctuation and the stronger interaction between fluctuations of closer amplitudes.

1. Introduction

The transition process in free shear layers such as wakes is very different from that in shear layers with solid boundaries. For instance, the transition process in a boundary layer along a solid wall consists of slow growth of a small amplitude fluctuation and abrupt production of high frequency components (Klebanoff, Tidstrom & Sargent 1962). On the other hand, the transition process in the wake behind a slender body consists of rapid growth of a small amplitude fluctuation and slow randomization of the fluctuation (Sato & Kuriki 1961; Sato 1970; Sato & Saito 1975). These facts suggest that a different approach is necessary for the artificial acceleration and deceleration of the transition process in different shear flows. The objective of the present investigation is to find sounds suitable for the control of the transition process in the wake behind a slender body. The control of transition of wakes may provide a better understanding of the mechanism of transition.

The transition region of the wake is divided into three subregions: the linear, nonlinear and randomizing regions. In the linear region a small amplitude fluctuation grows or decays exponentially in the flow direction depending on its frequency. The linear stability theory predicts the rate of growth as a function of frequency and the results agree very well with experimental observations (Sato & Kuriki 1961; Mattingly & Criminale 1972). The growth rate has a maximum at a frequency between lower and upper frequencies of neutral stability. This means that the linear region is equivalent to a band-pass filter. Small amplitude fluctuations existing in the wake are amplified selectively by the filter. The output of the filter is close to a sinusoidal fluctuation with the frequency of the maximum growth rate. When the amplitude of the fluctuation exceeds a threshold value, nonlinear interaction takes place. This leads to the generation of higher harmonics and the deformation of the mean-velocity distribution. In the randomizing region the fluctuation becomes more and more random and a turbulent wake is established.

If external sound is introduced, it induces a velocity fluctuation of small amplitude. If its frequency lies in the unstable region, the fluctuation grows exponentially. When the amplitude of the fluctuation becomes large, nonlinear interaction with the pre-existing natural fluctuation takes place. At present there are no satisfactory nonlinear theories, but some features of the nonlinear interaction have been clarified to some extent by both experiment and numerical computation (Sato & Saito 1975; Ko, Kubota & Lees 1970; Zabusky & Deem 1971). The nonlinear interaction results in the production of fluctuations whose frequencies are the sum and difference of the frequencies of the interacting fluctuations. The interaction between periodic and random fluctuations results in the randomization of a periodic fluctuation. In the present investigation we introduce periodic sounds or noise and try to find sounds appropriate for the control of the transition process.

2. Basic considerations

The three subregions of the transition region are illustrated schematically in figure 1. The streamwise variation of the wake width b and the fluctuation energy $\overline{u^2}$ is also shown. These data were taken from our previous experimental results. The linear region follows the laminar region and extends several wake widths. In this region the width remains almost unchanged and the energy increases exponentially. In the nonlinear region both b and $\overline{u^2}$ become a maximum, decrease and level off. It should be noted that the maximum of $\overline{u^2}$ does not correspond to the transition. The wave form of the strong fluctuation in the nonlinear region is almost periodic and the random component is very small. We find a kind of equilibrium part in which b and $\overline{u^2}$ remain almost unchanged. The growth of the random component takes place in the randomizing region, in which $\overline{u^2}$ decreases whereas b increases. The wave form becomes more and more irregular until the turbulence is established.

The linear growth rate in the wake has a maximum at f_m as shown in figure 2. In natural transition the random natural fluctuations grow with different growth rates at different frequencies and as a result an almost sinusoidal fluctuation with frequency f_m is formed at the end of the linear region. The intensity of natural fluctuations before the growth is of the order of 10^{-4} times the free-stream velocity U_0. The fluctuation is amplified up to around 10 % of U_0. When sound is introduced, a small amplitude velocity fluctuation is induced in the flow. This fluctuation grows everywhere in the wake but the largest growth takes place near the trailing edge. The stability calculation indicates that the growth rate is higher for a larger velocity defect (Mattingly & Criminale 1972; Nayaka 1976). Since the defect is largest near the trailing edge, the largest growth takes place there. The growth at other

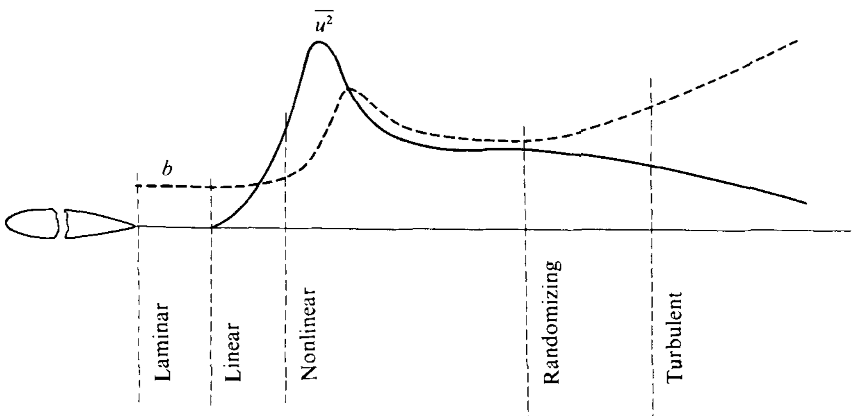

FIGURE 1. Subregions in transition region of wake. ---, wake half-width; ——, fluctuation energy.

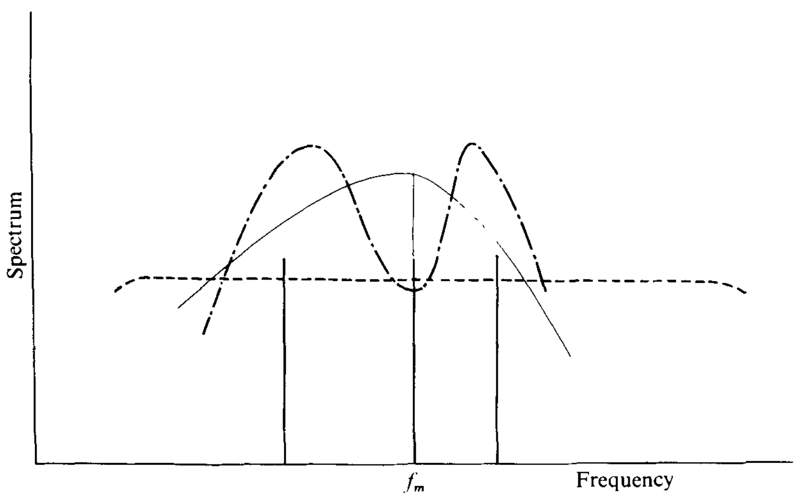

FIGURE 2. Spectra of sounds. ——, sinusoidal sounds; ---, white noise; —·—, two-peak noise; ——, linear growth rate.

places is negligible. Therefore the sound is equvalent to a localized disturbance introduced at the upstream end of the wake.

Concerning the nonlinear interaction of velocity fluctuations, two simple rules were deduced from our previous experiments. The first is concerned with the suppression of growth of one fluctuation by others. If there are two fluctuations with a substantial difference in amplitude, the growth of the weak fluctuation is suppressed by the strong fluctuation. The second rule is concerned with the production of components of other frequencies. The production of components whose frequencies are the sum and difference of those of two existing fluctuations is more effective when the amplitudes of the two fluctuations are close. These rules are purely empirical but they are useful in explaining observations. The control of transition is accomplished by the use of these rules, which hold not only for the interaction of periodic fluctuations but also for interaction between periodic and random fluctuations.

We chose the sound by considering the linear growth and the nonlinear interaction. Sound with frequency f_m was tried because the linear growth rate is large and strong growth suppression is expected. Sounds of frequencies higher or lower than f_m (shown in figure 2) induce fluctuations which grow with smaller growth rates. The amplitudes of these fluctuations are not so large and the nonlinear interaction may take place in a different manner. If the frequency is too far from f_m, the growth rate is extremely small and there may be no effect. We can use not only pure sound of one frequency but also sound with two or more frequencies. More components of different frequencies may lead to more effective nonlinear interaction. We can modulate sound by random noise. The modulation corresponds, according to a vortex model, to the perturbation of the vortex street in the wake. The strength of each vortex is perturbed by the amplitude modulation (AM) and the longitudinal spacing of the vortices is disturbed by the frequency modulation (FM). If the vortex street in the wake is unstable to such perturbations, these modulations might lead to early transition.

The transition seems to be accelerated by random noise. A simple noise is white noise, whose spectrum is flat as indicated in figure 2. The introduction of white noise is equivalent to increasing the level of natural fluctuations. Another noise is 'coloured' noise which has a peak at one frequency in the spectrum. By means of this noise we can place emphasis on the frequency. The noise with two peaks shown in the figure is a special noise which is very effective for accelerating the transition process. Details of this will be shown in §6.

The intensity of the sound is another important factor. The amplitude of the fluctuation directly induced by sound is very small. Therefore the fluctuation must grow considerably before it can affect the transition process. The growth is accomplished in the linear region. Because of the built-in high-gain amplifier in the wake, we do not need sound of high intensity. We expect a strong interaction when the amplitudes of the sound-induced fluctuation and the natural fluctuation are comparable.

From these considerations we chose the following sounds.

(i) Sinusoidal sound of frequency f_m.
(ii) Sinusoidal sound of frequency not f_m.
(iii) Sinusoidal sound of two frequencies.
(iv) Sinusoidal sound of four frequencies.
(v) Sinusoidal sound modulated by random noise (AM and FM).
(vi) White noise.
(vii) Noise with one peak in the spectrum.
(viii) Noise with two peaks.

3. Experimental arrangement

The experiment was conducted in the Low-Turbulence Wind-Tunnel of the Institute of Space and Aeronautical Science, University of Tokyo. The test section of the wind tunnel is 60 × 60 cm in cross-section and 3 m in length. There is a random residual fluctuation in the test section. The root-mean-square value of the natural fluctuation is about 0·05 % of the free-stream velocity. The arrangement for the experiment is shown in figure 3. A two-dimensional laminar wake was produced behind a thin

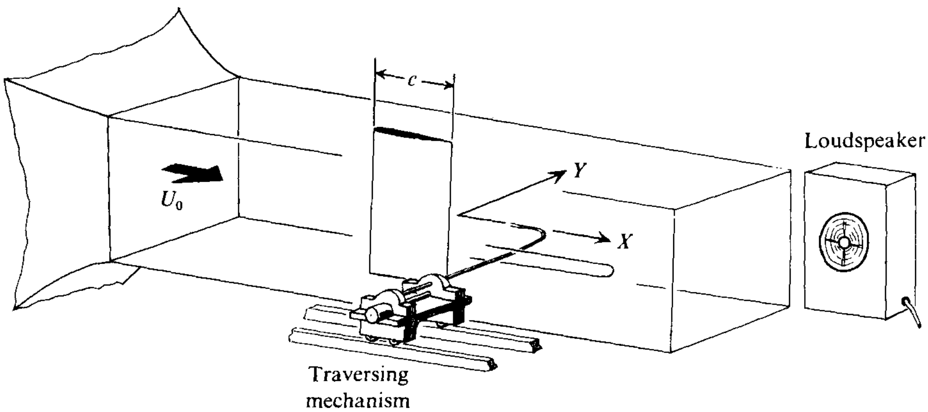

FIGURE 3. Experimental arrangement.

aerofoil placed parallel to the uniform flow. The aerofoil has a chord length c of 30 cm, a maximum thickness of 3 mm and a thickness at the trailing edge of about 0·2 mm. Details of the wind tunnel and the wake-producing aerofoil may be found in previous papers (Sato 1970; Sato & Saito 1975). The origin of the co-ordinate system is at the centre of the trailing edge and the X, Y axes are taken as shown in the figure. Experiments were carried out at three free-stream velocities: 5, 7 and 10 m/s. Since no qualitative differences were found at different wind speeds, detailed measurements were made at one speed, $U_0 = 10$ m/s. The Reynolds number $2U_0\delta/\nu$ based on the total thickness of the boundary layer at the trailing edge was about 4500. The mean and fluctuating components of the u velocity were measured by a single linearized constant-temperature hot-wire anemometer mounted on a traversing mechanism as shown in the figure. The output of the hot wire was recorded by an analog data recorder and processed by a hybrid analog–digital processor.

A loudspeaker of maximum power 10 W was placed at the downstream end of the test section and the sound from the loudspeaker was used as an artificial disturbance. Sound of one, two or four frequencies was used as a periodic disturbance. The amplitude modulation (AM) and the frequency modulation (FM) of the periodic sound were produced by modulators and a low frequency noise generator. The modulation ratio was fixed at unity; in other words each side band had a quarter of the power of the carrier wave. Noise with a flat spectrum between 30 and 2000 Hz was used as white noise. Coloured noise was produced by a band-pass filter. Noise with two peaks in the spectrum was obtained as the sum of two coloured noises.

The acoustic characteristics of the wind tunnel were investigated by traversing a small microphone in the test section. Resonant modes at various frequencies were found. They were obviously due to reflexions from the tunnel walls. Although sharp resonances were eliminated by sound-absorbing covers on the walls, some resonant modes still remained. We avoided the use of sounds at resonant frequencies. The intensity of off-resonant sound was uniform in three directions near the trailing edge of the aerofoil. Because the intensities of the velocity fluctuations induced by sound were different for different frequencies with the same loudspeaker input, we did not use the loudspeaker input as the indicator of the intensity of the disturbance. The intensity $(\overline{u_0^2})^{\frac{1}{2}}$ of the sound-induced velocity fluctuation at a point in the

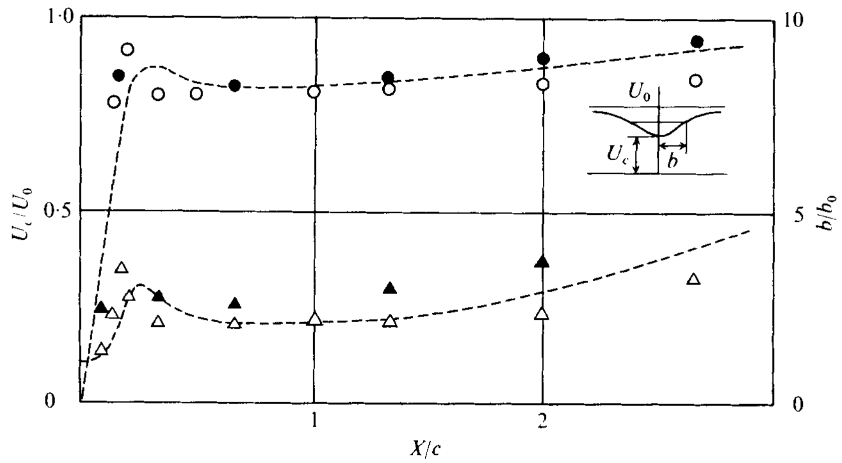

FIGURE 4. Streamwise variation of non-dimensional velocity on the centre-line U_c/U_0 and half-width b/b_0; $U_0 = 10$ m/s, $b_0 = 1{\cdot}1$ mm. ○, U_c/U_0 with sound with $f = f_m$; ●, U_c/U_0 with sound with $f = 0{\cdot}84 f_m$; △, b/b_0 with sound with $f = f_m$; ▲, b/b_0 with sound with $f = 0{\cdot}84f_m$; – – –, natural transition.

laminar part of the wake ($X = 10$ mm, $Y = 0{\cdot}6$ mm) was taken as the intensity of the artificial disturbance. If the frequency was fixed, $(\overline{u_0^2})^{\frac{1}{2}}$ was almost proportional to the loudspeaker input. Most of the measurements were made with $(\overline{u_0^2})^{\frac{1}{2}}$ equal to $0{\cdot}5\,\%$ of U_0. A change in the location of the loudspeaker or the direction of the incoming sound did not affect experimental results, if the induced velocity fluctuation was maintained at the same level.

4. Results with sound of one frequency

The velocity fluctuation induced by sound of one frequency is sinusoidal. At $U_0 = 10$ m/s fluctuations in the frequency range 400–800 Hz grow exponentially in the linear region ($X = 10 \sim 25$ mm) and the maximum growth occurs at 600 Hz. Sound whose frequency is outside this range has no effect on transition. The amplitude of a fluctuation of large growth rate becomes large enough to suppress the growth of the natural fluctuation and the wave form of the fluctuation is purely sinusoidal. If the frequency of the sound is much different from f_m, the amplitude does not become large enough for the suppression of growth of the natural fluctuation. In this case two fluctuations co-exist and the wave form is irregular. Thus we distinguish two kinds of periodic sound: one of the frequency close to f_m and the other of the frequency far from f_m but between 400 and 800 Hz. The effects of these two sounds on the transition process are entirely different. Figure 4 shows the streamwise variation of the velocity U_c on the centre-line and the half-width b of the wake in the presence of sounds of different frequencies. U_c and b are non-dimensionalized by the free-stream velocity U_0 and the width b_0 at $X = 0$, respectively. Results for natural transition are added as broken lines. The maxima of U_c and b found at around $X/c = 0{\cdot}3$ are due to a strong nonlinear interaction with the mean velocity. Both decrease between $X/c = 0{\cdot}3$ and $0{\cdot}6$ and increase gradually at $X/c > 1$. It is impossible to pinpoint the downstream end of the transition region from these data but the values of

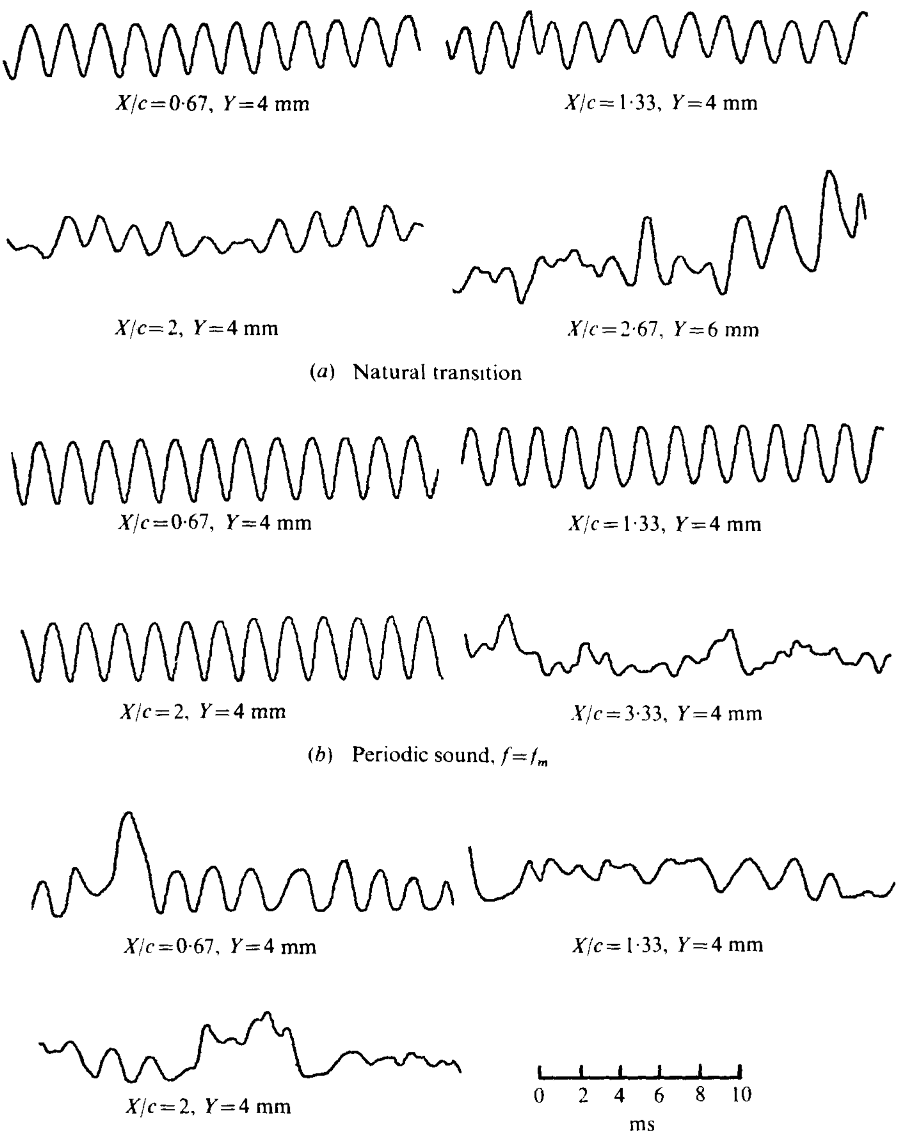

FIGURE 5. Wave forms of u fluctuation. Time increasing from left to right.

U_c/U_0 and b/b_0 at $X/c > 0{\cdot}6$ are smaller for $f = f_m$ (600 Hz) and larger for $f = 0{\cdot}84f_m$ than those for natural transition. This indicates the deceleration and acceleration of the transition process, respectively. A more direct indicator of the transition process is the wave form. Figure 5 shows wave forms of the u fluctuation for three cases: natural transition, sound with $f = f_m$ and sound with $f = 0{\cdot}88f_m$ (f = 528 Hz). At $X/c = 0{\cdot}67$ the wave forms for natural transition and with sound with $f = f_m$ are regular and periodic. With sound with $f = 0{\cdot}88f_m$ the wave form is somewhat irregular at the same X station. The wave forms change downstream and at $X/c = 2$ the wave forms in the three cases are clearly different. With sound with $f = f_m$ the

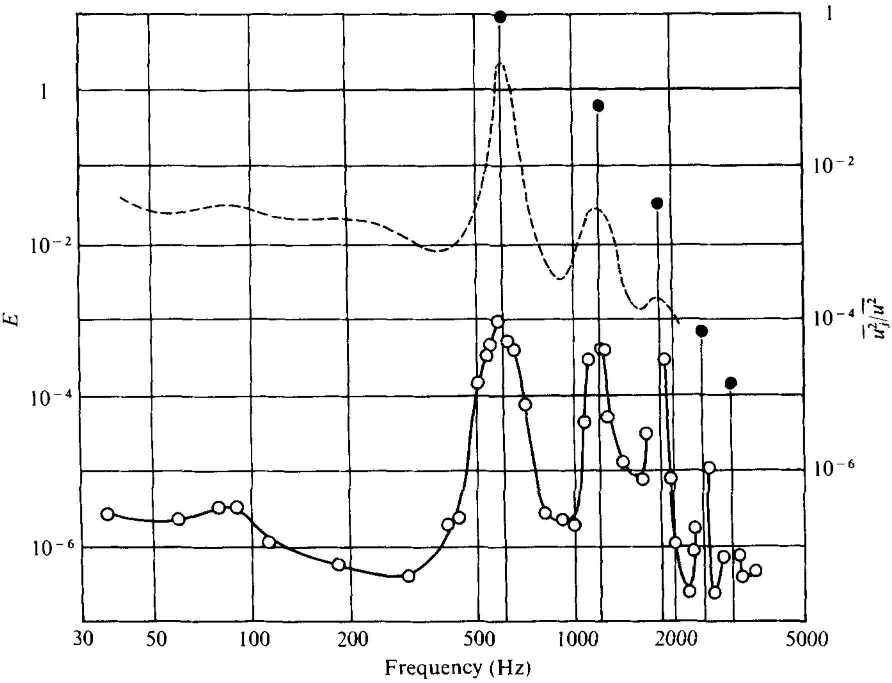

FIGURE 6. Energy spectrum at $X = 150$ mm ($X/c = 0{\cdot}5$), $Y = 4$ mm with sound with $f = f_m$. ○, continuous part E, arbitrary scale; ●, discrete components $\overline{u_j^2}/\overline{u^2}$; ---, natural transition.

wave form remains periodic, but with $f = 0{\cdot}88f_m$ the wave form is very irregular. The wave form for natural transition lies between the two. It is clear that the two sounds with different frequencies have opposite effects on the transition process.

Figure 6 shows the energy spectrum with sound with $f = f_m$. Open circles indicate the continuous part E and closed circles the discrete components $\overline{u_j^2}$. The scale of E is arbitrary and $\overline{u_j^2}$ is shown as a fraction of the total energy $\overline{u^2}$. The value of $\overline{u_1^2}/\overline{u^2}$ at $f = 600$ Hz is almost unity, which means that the fluctuation is periodic with that frequency. There are weak higher harmonics up to $j = 5$. The spectrum for natural transition is shown as a broken line. The relative magnitude of the two continuous spectra is expressed correctly. We find a remarkable reduction of the continuous part. This is obviously due to the nonlinear growth suppression by the strong 600 Hz component. Figure 7 shows the spectrum with sound with $f = 0{\cdot}84f_m$. In this case the continuous part at low frequencies is quite strong and the discrete 504 Hz component is only 30 % of the total energy $\overline{u^2}$ at $X/c = 0{\cdot}67$. This indicates that the growth rate of the 504 Hz component is small and there is no growth suppression by that component. This co-existence of the 504 Hz component and the natural, continuous component is a favourable situation for a strong mutual interaction which results in fast randomization of the periodic component. The spectrum at $X/c = 1{\cdot}33$ is continuous and flat, whereas in natural transition there is still a peak around 600 Hz. The acceleration of the transition process is obvious. In figure 8, which is for a larger X ($X/c = 2{\cdot}67$), the spectrum with sound with $f = f_m$ still has several

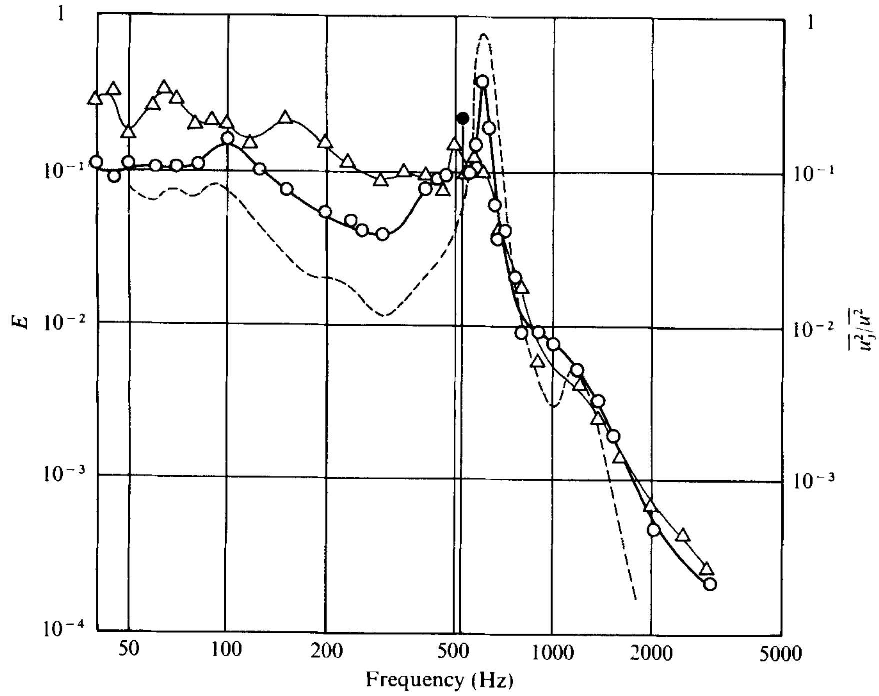

FIGURE 7. Energy spectra with sound with $f = 0{\cdot}84 f_m$. ○, continuous part E at $X = 200$ mm ($X/c = 0{\cdot}67$), $Y = 4$ mm; ●, discrete components $\overline{u_j^2}/\overline{u^2}$ at $X = 200$ mm, $Y = 4$ mm; △, continuous part E at $X = 400$ mm ($X/c = 1{\cdot}33$), $Y = 4$ mm; ---, natural transition at $X = 40$ mm, $Y = 4$ mm.

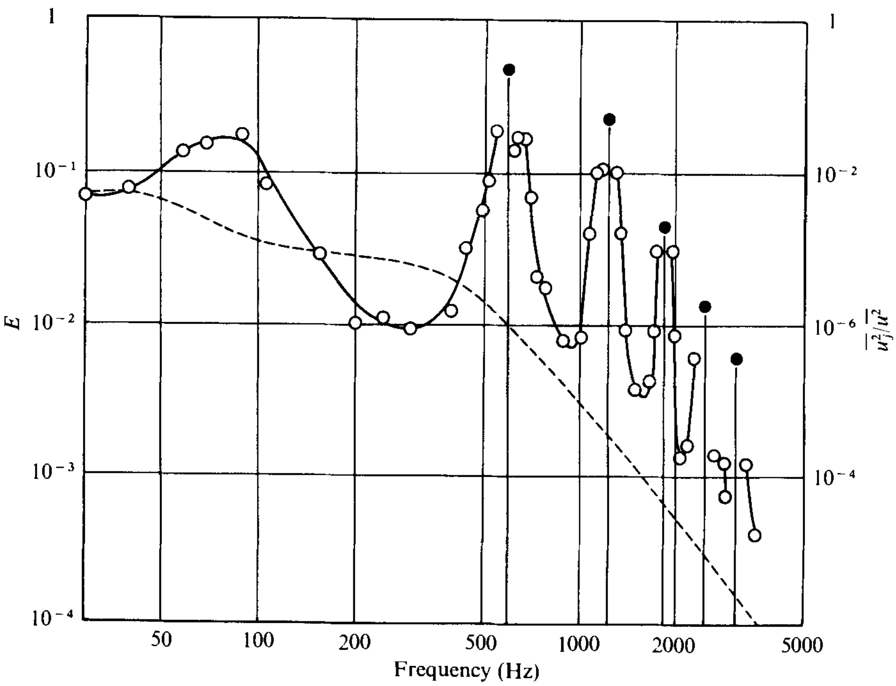

FIGURE 8. Energy spectrum at $X = 800$ mm ($X/c = 2{\cdot}67$), $Y = 4$ mm with sound with $f = f_m$. Notation as in figure 6.

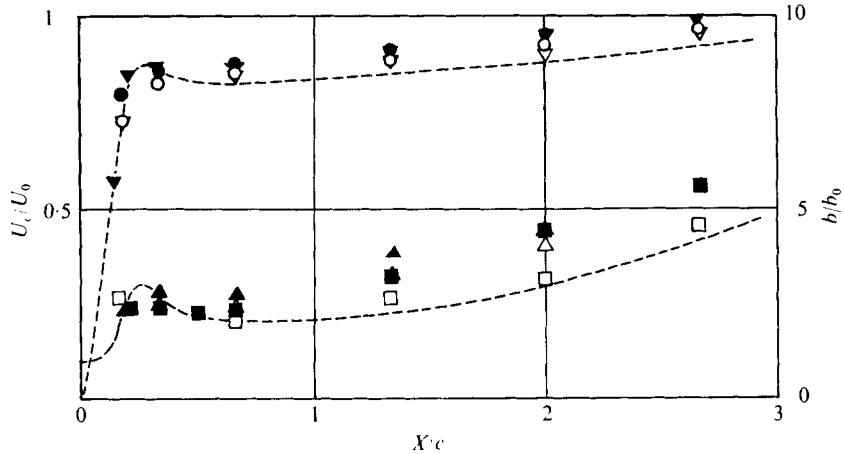

FIGURE 9. Streamwise variation of non-dimensional velocity on the centre-line and half-width. ○, U_c/U_0 with sound with $f = 0{\cdot}94f_m$ and $1{\cdot}09f_m$; ●, U_c/U_0 with sound with $f = 0{\cdot}81f_m$, $0{\cdot}91f_m$, $1{\cdot}08f_m$ and $1{\cdot}22f_m$, frequency modulated; △, b/b_0 with sound of two frequencies; ▲, b/b_0 with FM sound of four frequencies; ▽, U_c/U_0 with white noise; ▼, U_c/U_0 with two-peak noise; □, b/b_0 with white noise; ■, b/b_0 with two-peak noise; ---, natural transition.

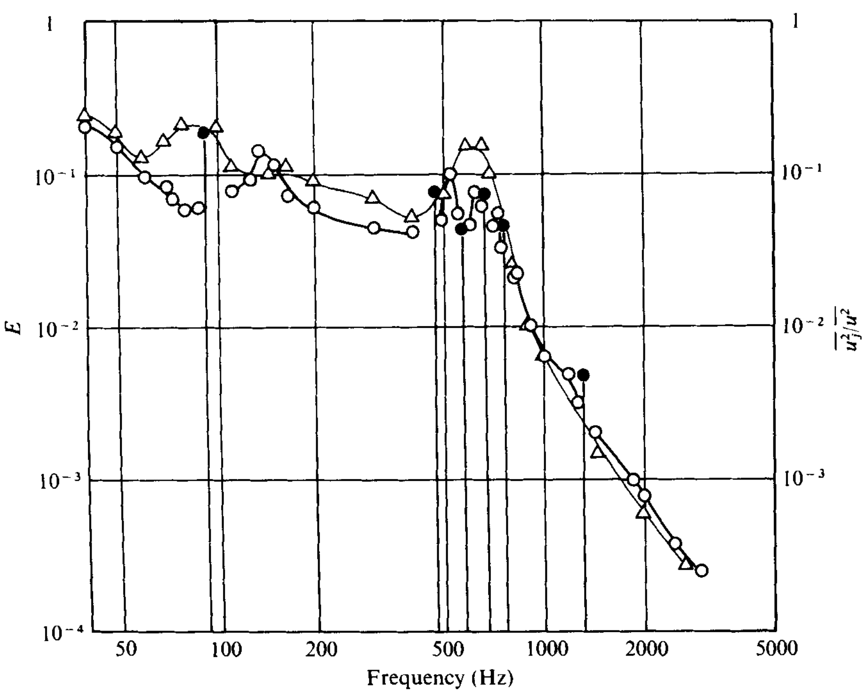

FIGURE 10. Energy spectra at $X = 400$ mm ($X/c = 1{\cdot}33$), $Y = 4$ mm. ○, continuous part E with sound with $f = 0{\cdot}94f_m$ and $1{\cdot}09f_m$; ●, discrete component $\overline{u_j^2}/\overline{u^2}$ with sound with $f = 0{\cdot}94f_m$ and $1{\cdot}09f_m$; △, continuous part E with sound with $f = 0{\cdot}81f_m$, $0{\cdot}91f_m$, $1{\cdot}08f_m$ and $1{\cdot}22f_m$, frequency modulated.

discrete peaks but for natural transition a smooth turbulence spectrum has been established.

We summarize the results for periodic sound of one frequency as follows. Sound of frequency close to f_m is effective in decelerating the transition process. The closer the frequency is to f_m, the greater is the effect. If the frequency of the sound is considerably different from f_m the effect is reversed, giving acceleration. With sound of frequency too different from f_m there is no effect.

5. Results with periodic sound of two or more frequencies

We expect more effective nonlinear interaction with sound of more frequencies. If sound of two different frequencies is introduced, we have three interacting components (two artificial and one natural fluctuation). This accelerates the transition process. We can use a sound with $f < f_m$ and one with $f > f_m$. In fact we mixed two sinusoidal signals before they were converted into sound. We used also sound of four frequencies: two with $f < f_m$ and two with $f > f_m$. Figure 9 shows the streamwise variation of U_c/U_0 and b/b_0 for sound of two and four frequencies. The increase in U_c/U_0 and b/b_0 at $X/c > 1$ clearly shows that sound of two frequencies accelerates the transition. Four frequencies are more effective. Figure 10 shows the spectrum in the presence of sound with $f = 0{\cdot}94 f_m$ and $1{\cdot}09 f_m$ at $X/c = 1{\cdot}33$. Besides two discrete components at 564 Hz and 657 Hz, a 93 (= 657 − 564) Hz, a 1221 (= 657 + 564) Hz, a 471 (= 564 × 2 − 657) Hz and a 751 (= 657 × 2 − 564) Hz component are found. The number of such discrete components increases downstream owing to nonlinear interactions. An effective randomization takes place through the interaction of these components and the continuous natural fluctuation.

With a sound with four frequencies we obtain a similar spectrum. If we choose the combination of four frequencies and amplitudes properly, we have many more discrete components and much more effective randomization. Figure 10 shows the spectrum with sound of four frequencies ($0{\cdot}81 f_m$, $0{\cdot}91 f_m$, $1{\cdot}08 f_m$ and $1{\cdot}22 f_m$) each frequency modulated by low frequency noise. Because of the modulation the spectrum is not discrete but continuous. If we compare two spectra in figure 10 both for the same point in the wake, we find more acceleration by the FM sound of four frequencies. The difference is not great, however. The spectrum with AM sound of four frequencies is almost the same. It seems that most of the randomness necessary for the transition is provided by the natural fluctuation and that an artificial randomness introduced by the modulation constitutes only a small addition to the natural randomness. We conclude that sound of more frequencies is more effective in accelerating the transition process. The effect of modulation of the sound by random noise is small.

6. Results with random noise

The simplest random noise is white noise with a flat spectrum. When white noise is introduced, spectral components between around 450 and 800 Hz grow in the linear region and components outside this frequency range do not. They are just wasted. Therefore 'coloured' noise with frequencies lying in the above frequency range is more effective with less power. Noise with a peak at f_m was found to be more

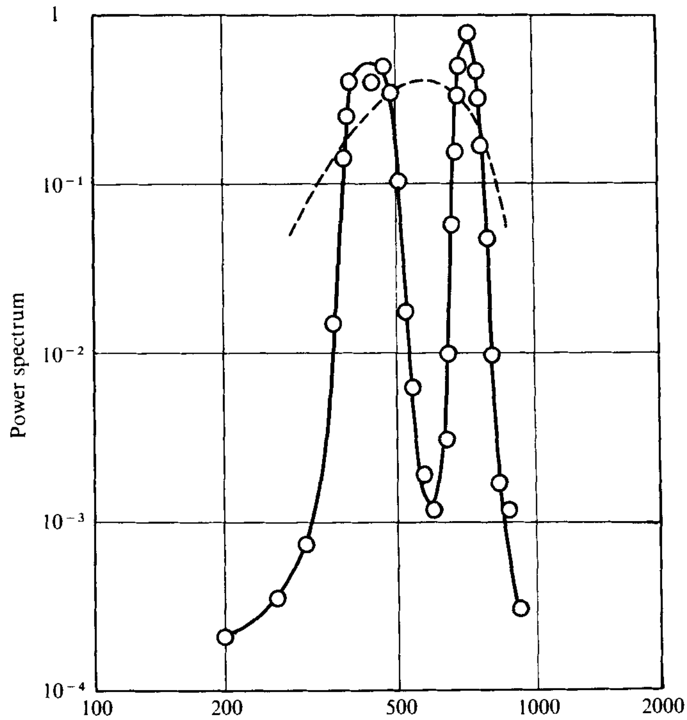

FIGURE 11. Power spectrum of two-peak noise. ---, schematic representation of the linear growth rate.

efficient than white noise in accelerating the transition process. Because of the selective growth in the linear region we tried a special noise. A strong nonlinear interaction may take place if there are many spectral components of equal intensity at the downstream end of the linear region. Such a flat spectrum is produced by the noise whose spectrum is inverse to the linear amplification. Figure 11 shows the power spectrum of such an artificial noise. It has two peaks around 450 and 700 Hz and a minimum at 600 Hz ($= f_m$). The broken line is a schematic representation of the linear growth rate. We expect a flat energy spectrum of velocity fluctuations between 450 and 700 Hz at the end of the linear region. The intensities of the two peaks and the depth of the central valley were determined purely experimentally. This noise resulted in a wide and almost flat energy spectrum as will be shown later.

Figure 9 shows the streamwise variation of U_c/U_0 and b/b_0 in the presence of white noise and two-peak noise. In both cases a rapid increase of U_c and an early widening of the wake are clearly observed. The white noise is effective but the two-peak noise performs much better. With the two-peak noise the width of the wake at $X/c = 2$ is about 50 % more than that in natural transition. Figure 12 shows the energy spectrum at $X/c = 1{\cdot}33$ in the presence of white noise. The difference from the spectrum of natural transition, shown by a broken line, is small. A peak at 600 Hz is the result of the selective growth in the linear region. Figure 13 shows a spectrum at $X/c = 0{\cdot}1$ in the presence of two-peak noise. This location roughly corresponds to the downstream end of the linear region. On comparing with natural transition we observe

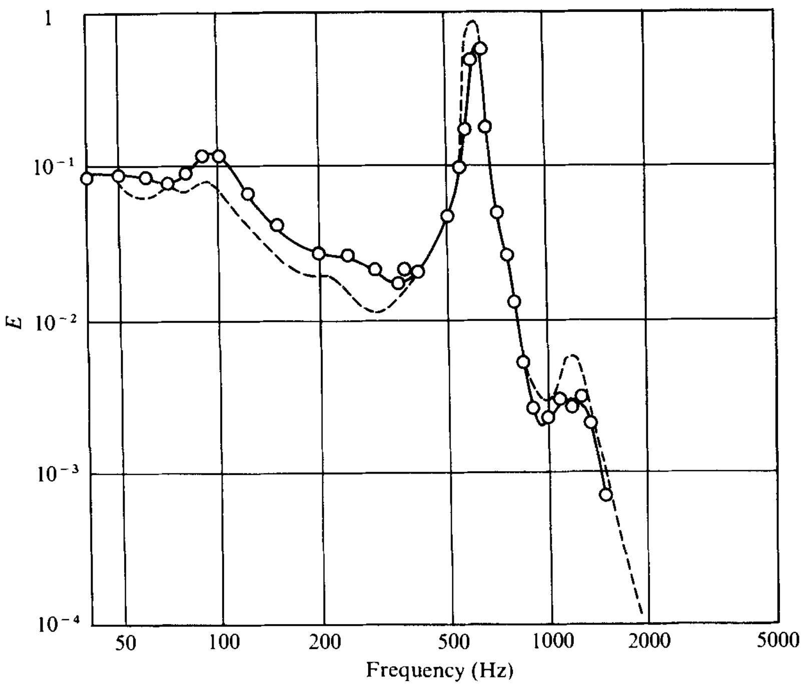

FIGURE 12. Energy spectrum at $X = 400$ mm ($X/c = 1{\cdot}33$), $Y = 4$ mm with white noise. ---, natural transition.

a wide and flat spectrum around 600 Hz. The spectrum around 1200 Hz (second harmonics) is also wide and flat. With this spectrum at $X/c = 0{\cdot}1$ the acceleration of the transition process is obvious as shown in figure 14. Sharp peaks in natural transition are flattened and there is more energy in the low frequency region. The spectrum is very close to that of a turbulent wake at this small $X/c = 0{\cdot}5$. The spectrum at a much larger X/c of $1{\cdot}33$ in the presence of white noise (figure 12) is more peaked than figure 14. The energy spectrum at $X/c = 1{\cdot}33$ with sound of four frequencies (figure 10) is close to figure 14. We conclude that the two-peak noise is so far the most effective in accelerating the transition process.

7. Discussion

The transition process is illustrated very well by the energy spectrum. When the line spectrum, a hump or valley in the continuous spectrum disappears, we conclude that the transition has been completed and turbulence has been established. It is possible to consider the situation in which regular fluctuations with slightly different frequencies appear intermittently and as a result of the long-time average a smooth spectrum is obtained. Some visualization work in the wake supports the picture. In this case, from hot-wire measurements, we expect a strong fluctuation in the spectrum averaged over a short time. The transitional spectrum actually fluctuated pretty much but the smooth, fully developed spectrum was quite steady. The statistical treatment of the short-time spectrum is a good subject for future work.

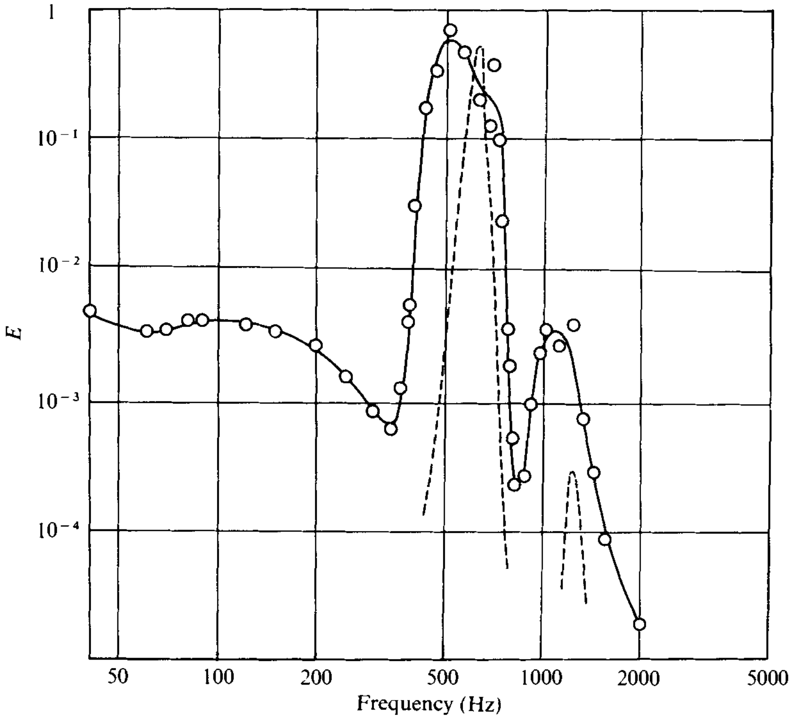

Figure 13. Energy spectrum at $X = 30$ mm ($X/c = 0{\cdot}1$), $Y = 1$ mm with two-peak noise. ---, natural transition.

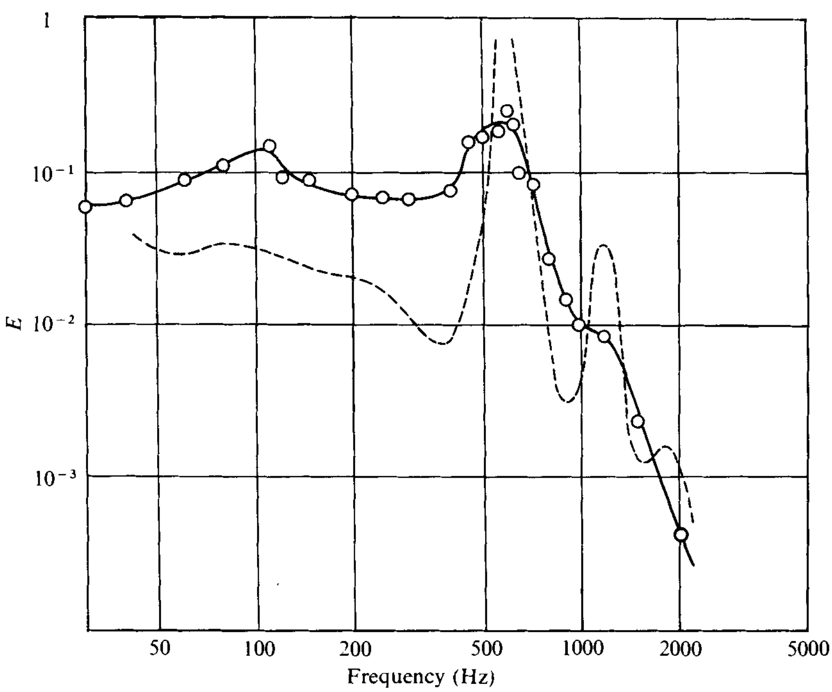

Figure 14. Energy spectrum at $X = 150$ mm ($X/c = 0{\cdot}5$), $Y = 4$ mm with two-peak noise. ---, natural transition.

The wave form of the fluctuation is also a good indicator of the transition process. The periodic or random nature is clearly observed in the wave form. On the other hand, the fluctuation energy $\overline{u^2}$ does not indicate the transition. As shown in figure 1, the energy increases rapidly in the linear region, reaches a maximum and gradually decreases. The maximum value of $\overline{u^2}$ does not correspond to the establishment of turbulence. The transition takes place while $\overline{u^2}$ decreases monotonically. The half-width b and the centre-line velocity U_c are good indicators. They increase and decrease at small X, stay almost constant in the nonlinear region and increase again as the randomization process proceeds. The rapid increase in b or U_c corresponds to the establishment of turbulence.

Concerning the intensity of the sound we distinguish three cases. If the sound is very weak, the sound-induced fluctuation does not grow to an intensity comparable to that of the pre-existing natural fluctuation. In that case the nonlinear growth of the sound-induced fluctuation is suppressed by the strong natural fluctuation and the sound has little effect. The present investigation was made with sound of moderate intensity. In this case, if the linear growth rate is high the artificial fluctuation, whose amplitude is large, suppresses the growth of the natural fluctuation. This leads to the deceleration of the transition process. If the growth rate is moderate, the artificial fluctuation grows to the same level as the natural fluctuation. Their interaction is then very effective and the transition process is accelerated. The intensity of sound required for effective interaction depends on the intensity of the natural fluctuation. If the natural fluctuation is high, the sound must be strong, but if the natural fluctuation is low, a weak sound is enough for the control of transition. An extremely strong sound seems to be less effective because no artificial fluctuation can grow beyond a certain threshold value.

In the low-turbulence wind tunnel there is no low frequency fluctuation near the trailing edge. Low and high wavenumber components are produced by interactions among 'middle' wavenumber components which have grown in the linear region. We expect more effective interaction with more interacting components. In that respect sound with more frequencies is more effective in accelerating the transition process. On the other hand, a strong interaction is expected when the amplitudes of interacting components are close. Therefore the intensities of sounds of various frequencies must be so adjusted that this occurs. If the frequencies and intensities are properly chosen, sound of four frequencies is very effective in accelerating the transition process. Two-peak noise has many more interacting components and it is the most effective.

The interaction among periodic fluctuations results in the production of further periodic fluctuations. Therefore, if there is no random fluctuation, a turbulent state may never be realized. At the beginning of the experiment, we expected that the introduction of artificial randomness might be useful for achieving early transition. Amplitude or frequency modulation of sound by random noise was expected to be effective but experimental results indicated a small effect of the modulation. Obviously, there is a natural 'seed' of randomness: the residual turbulence in the wind tunnel. Although it is very weak at the beginning, it grows to a large amplitude. Because of the large growth, a small difference in the initial value does not make much difference. In fact, the delay in transition produced by the reduction of residual turbulence is not great. In the same sense, the introduction of artificial randomness by modulation has a limited effect. A noticeable delay in transition is accomplished only by the suppression of the growth of random fluctuations by a large amplitude periodic fluctuation.

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